6. The angle increases 10° a year.
7. Mag. from Cor. D.M. Estimated 8·0. Is 8·7 in C.P.D. and marked: Angle decreasing, distance decreasing.
   There is a 13th mag. star 200° ±, 35″ ±.
8. Change. Two careful estimations with the chief star out of the field.

Short Method for the Calculation of the Orbits of Celestial Bodies.

By D. A. Pio.

(*Communicated by the Secretaries.*)

1. *Gain of Time is the Object.*—The determination of the elements of the orbits described by a celestial body has been carried by Gauss to the highest pitch of perfection the present state of mathematical science allows us to reach. The author has not the pretension to propose a new method for solving the same problem; he only intends to substitute short, simple, and easy calculations for those which the method of Gauss requires, and limits himself to the most simple case: how to determine the first approximate values of the elements of a newly discovered celestial body with the least loss of time, so as to render the calculation of a new orbit an easy performance.

On the authority of the illustrious French mathematician M. H. Poincaré, the writer begs to state that the calculation of the first approximate orbit, by Gauss's method, requires from fifteen to seventeen hours. A reduction to about six to seven hours is what the author is working for. A gain of about eleven hours with a sufficiently approximate determination of the elements would be a service to practical astronomers.

The author proposes, for attaining this short calculation of orbits, a modification of Gauss's method containing innovations of great practical interest for the efficient simplification of the mathematical work.

The writer begs to remark that the problem of orbits has been really solved long ago; that the question is only how to render tedious calculations more easy, but not to present new theories by which the work of La Place, Olbers, and Gauss may be swept away. This is the point of view from which the author begs his work to be considered. Practical improvement, not radical subvertment, is the only amelioration Gauss has left posterity to accomplish.

2. *Preliminary Remarks.*—It is supposed that the data furnished by the observations and those taken from the solar tables are corrected for precession, nutation, and aberration of the fixed stars. Parallax and aberration in time are not taken into consideration. The calculations are made with logarithms to not more than five decimal places.

3. *Simplifications of Gauss's Method.*—(1) Gauss established the equation

$$\mathfrak{M} . \sin^4 z = \sin (z+\omega)$$

which leads to the value of r_2. This part of the solution cannot be improved. However, modern authors have brought confusion into the signs (+ or −) of the many auxiliary quantities occurring in the calculation: especially the case where r_2 is *less* than unity offers difficulties. The writer has reverted to the *Theoria Motus*, and has removed all ambiguities (see articles 4 and 5).

(2) The solution of the equation

$$\mathfrak{M} . \sin^4 z = \sin (z+\omega)$$

has been rendered shorter by using logarithms with three decimals for the first approximations. The method of logarithmic differences has been reduced to its simplest form (see articles 7 and 8).

(3) Gauss deduces the elements of the orbit from the known values of r_1 and r_3, and from the angle $2f' = v_3 \sim v_1$ contained between them. The calculation of these quantities requires the previous calculation of the distances from the Earth, ρ_1 and ρ_3, which as done in the usual manner is the most lengthy and the most tedious part of the whole work. The usual formulæ, as given in Tisserand, *Leçons sur la Détermination des Orbites*, pp. 76 and 77, are twenty-four in number, and some are very troublesome. The writer has reduced them to six by using the "fundamental equations" for the calculation of $\mathfrak{r}_1 = \rho_1 \cos \beta_1$ and $\mathfrak{r}_3 = \rho_3 \cos \beta_3$. The time thus gained amounts to some hours (see article 9).

(4) The parameter, the eccentricity, and the longitude of the perihelium of an orbit are the most difficult points in the determination of the elements. Gauss invented a most ingenious method for the calculation of the parameter by successive approximations through auxiliary tables. The author has much shortened the calculation of the parameter by reverting to the formula of Euler given in *Theoria Motus*, § 86 (VII.). It is the shortest existing.

(5) The formulæ usually employed for the calculation of ϕ, the angle of eccentricity, and the true anomalies v_1 and v_3 are very tedious. They occasion a considerable loss of time (see Tisserand, p. 80). The arc v_1 has been determined by a simple formula taken from *Theoria Motus*. The value of the eccentricity may be calculated very easily when the parameter, one of the radii vectores, and the corresponding true anomaly are known. The author, profiting by this circumstance, calculates *two* separate values for e and takes their *geometrical mean* as the most probable value.

Each of the above modifications is certainly a detail of no theoretical importance by itself, and has no other merit than

either to shorten the calculation or to increase the accuracy. The author begs to submit that all these modifications, taken as a whole, give quite a new appearance to the method of Gauss.

The question is, then, whether the above modifications satisfy or not the requirements of practical astronomers.

4. *Rules of Signs.*—The calculations preliminary to the solution of

$$\mathfrak{M} \, . \sin^4 z = \sin (z + \omega)$$

are relative to the determination of the quantities ψ, δ, τ, T, Σ, σ, S, d, u, ω, Ω, and h. The angles ψ, δ, τ, and u are to be taken always as positive arcs *less* than 180°. The arc σ is always less than 90° : it may be positive or negative, but it *must* be the *least* of all values that result from the absolute number found for $tg \, (\sigma + \delta)$; and the sign of $tg \, (\Sigma = \delta + \sigma)$ must be determined in conformity. The arc ω must always be less than 90°, but it may be negative or positive. The signs of the linear quantities T, S, d, Ω, and h are + or − according as the calculation determines them. Every other rule, even given by great authorities, leads to impossible results.

Note.—Oppolzer's rule to make T always positive is false when r_2 is less than 1. In this last case ω also is positive and not negative.

5. *A Difficulty in the Solution of* $\mathfrak{M} \, . \sin^4 z = \sin (z + \omega)$.—The equation

$$\mathfrak{M} \, . \sin^4 z = \sin (z + \omega)$$

is usually applied only to the planetoids where r_2 is comprised between 1·5 and 4, and then ω is usually negative and nearly equal to z. In such cases the unknown value of z is found by successive approximations, the value of $-\omega$ being the first, and the method of logarithmic differences leads very rapidly to a value sufficiently approximate for the first determination of the orbit. However, when r_2 is less than R_2, then δ is greater than 90°, and the angle ω is not only positive but very different from z, so that it becomes difficult to find a first approximation for z. As usually ω is small, by assuming $\sin \omega = 0$ the equation

$$\mathfrak{M} \; \sin^4 \omega = \sin (z + \omega)$$

becomes

$$\mathfrak{M} \, . \sin^3 z = \cos \omega$$

and then

$$\sin z = \sqrt[3]{\frac{\cos \omega}{\mathfrak{M}}}$$

which may serve as first approximation.

6. *Logarithms to three Decimal Places.*—In the usual manner of calculating the orbits much time is wasted by using logarithms to many decimal places, while the results found out with so

much labour are scarcely trustworthy to the third figure. This happens with the first attempts to solve the equation

$$\mathfrak{M} . \sin^4 z = \sin (z + \omega)$$

Also the correction of the approximate value of the parameter (Euler's formula) needs only logarithms to three decimal places.

7. *Logarithmic Differences.*—The method of logarithmic differences given by Gauss in *Theoria Motus*, § 11, for the solution of Kepler's Problem is excellent for solving the equation

$$\mathfrak{M} . \sin^4 z = \sin (z + \omega)$$

Its shortest form is that the author proposes.

EXAMPLE.—N[1·0822321] . $\sin^4 z = \sin(z - 12^\circ)$, where $\mathfrak{M}$ = N[1·0822321], $\omega = -12^\circ$, λ = logarithmic difference of $\sin z_2$ for one minute (respectively one second), μ = log dif. for 1′ (or 1″) of $\sin (z_2 + \omega)$, $\Delta = \log \sin^4 z_2 - \log \sin^4 z_1$, $z_1 = -\omega$.

$\log \sin z_1$	9·318	$\lambda = 0·5$		$\log \sin^4 z_2$	9·362
	4				4
$\log \sin^4 z_1$	7·272	$\mu = 5·6$		$\log \sin^4 z_2$	7·448
$\log \mathfrak{M}$	1·082	$\mu - 4\lambda = 3·6$		$\dfrac{\Delta}{\mu - 4\lambda} =$	49′
$\log \sin (z_2 + \omega)$	8·354	$\Delta = 176$			
$z_2 + \omega = 1^\circ\ 17'·5$		$\omega = -12^\circ$		$z_2 =$	$13^\circ\ 17'·5$
				$z_3 =$	$14^\circ\ 6'·5$

Six lines with logarithms are sufficient.

8. *Calculation of* $\rho_1 . \cos \beta_1$ *and* $\rho_3 . \cos \beta_3$.—The condition that the centre of the Sun must lie in the plane determined by the three observed positions of the celestial body furnishes three "fundamental equations," (5), (6), (7), in Tisserand, p. 47, containing the unknown quantities $\mathfrak{r}_1 = \rho_1 . \cos \beta_1$, $\mathfrak{r}_2 = \rho_2 . \cos \beta_2$, $\mathfrak{r}_3 = \rho_3 . \cos \beta_3$. One of them, $\mathfrak{r}_2$, being known, the determination of $\mathfrak{r}_1$ and $\mathfrak{r}_3$ becomes very easy. Equation (7) furnishes directly $\mathfrak{r}_1$, and from (6), where $\mathfrak{r}_1$ and $\mathfrak{r}_2$ are known, $\mathfrak{r}_3$ is easily found.

9. *Concluding Remarks.*—(1) The abridgments introduced by the author will furnish the values of the elements with sufficient accuracy, when the intervals of time between two successive observations are not more than ten days for planetoids.

(2) The purpose of a first determination of an orbit being first the calculation of a provisory ephemeris, and next the setting-up of numerical values for the elements approximate enough to allow the easy correction of these numerical values by comparison with a series of observations, any luxury of calculations, as those indicated in the text-books, is a mere waste of time. Whatever be the care bestowed upon the first determination of an orbit, the result arrived at by so much work will be *discarded*. Therefore brevity ensuring sufficient accuracy is the ideal of the first determination of an orbit.

(3) The calculation of an orbit is considered as the trial-piece of an incipient astronomer. It is, indeed, a proof of proficiency, especially if carried out by the method of Gauss, as explained in Oppolzer's text-book. But even the dreaded method of Gauss may be popularised, and that is what the author has attempted. In about six to seven hours, when the observations have been already reduced and corrected, to determine with sufficient accuracy the numerical values of the elements of an orbit without employing special auxiliary tables, only using the common logarithmic tables of numbers and trigonometrical functions to no more than five places of decimals, this is certainly no achievement in theory, but only a sort of mechanical craft. However, is it useful or not? Has the writer succeeded in building up an engine of calculation proper for the use of practical astronomers?

10. *A Test for Asteroids.*—The writer has chosen as an example for the application of the simplified method of Gauss the same observations from which Gauss himself deduced the orbit of *Juno* in *Theoria Motus*, § 150 to § 155. His calculations are a model of accuracy, and a comparison with those made by the writer shows the degree of precision the first values for the elements may reach when the modifications proposed by the author are used. The following table contains in its first column the results of the "last hypothesis" of Gauss:

	Gauss.	Pio.	Differences.
r_2	2·1183	2·1187	+0·0004
$2f'$	7° 34′ 54″	7° 35′ 7″	+13″
a	2·6450	2·6499	+0·0049
e	0·24532	0·24744	+0·00212
i	13° 6′ 44″	13° 7′ 30″	+46″
☊	171° 7′ 49″	171° 8′ 29″	+40″
π	52° 18′ 9″	52° 30′ 23″	+12′ 14″
m''	824″·80	822″·50	−2″·3
L_0	41° 52′ 22″	41° 58′ 22″	+6′ 0″

Formulæ.

1. *Symbols.*—For the three observations of the planet at the epochs E_1, E_2, and E_3, we have:

$E_2 - E_1 = t_1$.
$E_3 - E_2 = t_2$.
R_1, R_2, R_3, radii vectores of Earth.
L_1, L_2, L_3, geocentric longitudes of Sun.
λ_1, λ_2, λ_3, geocentric longitudes of planet.
β_1, β_2, β_3, geocentric latitudes of planet.
l_1, l_2, l_3, heliocentric longitudes of planet.
b_1, b_2, b_3, heliocentric latitudes of planet.

r_1, r_2, r_3, radii vectores of planet.
v_1, v_2, v_3, true anomalies of planet.
ρ_1, ρ_2, ρ_3, distances of planet from Earth.
e, eccentricity of planet.
p, half parameter of planet.
a, half major axis of planet.
T^d, planet's revolution in mean solar days.
m'', its mean diurnal motion in seconds of arc.
i, inclination of planet's orbit.
Ω, longitude of planet's ascending node.
Π, longitude of planet's perihelium.
L_0, mean longitude of planet at epoch E_0.
E_0, epoch of departure.

2. *Preliminary Calculations.*—

$$A = R_1[tg\beta_3 . \sin(\lambda_1 - L_1) - tg\beta_1 . \sin(\lambda_3 - L_1)] \quad \ldots \quad (1)$$

$$B = R_3[tg\beta_3 . \sin(\lambda_1 - L_3) - tg\beta_1 . \sin(\lambda_3 - L_3)] \quad \ldots \quad (2)$$

$$C = R_2[tg\beta_3 . \sin(\lambda_1 - L_2) - tg\beta_1 . \sin(\lambda_3 - L_2)] \quad \ldots \quad (3)$$

$$D = -R_2[tg\beta_3 . \cos(\lambda_1 - L_2) - tg\beta_1 . \cos(\lambda_3 - L_2)] \quad \ldots \quad (4)$$

$$tg . \psi = \frac{tg\beta_2}{\sin(\lambda_2 - L_2)} \quad \psi \text{ positive and } < 180^\circ \quad \ldots \quad \ldots \quad (5)$$

$$tg\delta = \pm \frac{tg(\lambda_2 - L_2)}{\cos . \psi} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (6)$$

NOTE.—$\delta > 90^\circ$ when $(\lambda_2 - L_2) \begin{matrix} < 90^\circ \\ > 270^\circ \end{matrix}$; $\delta < 90^\circ$ when $(\lambda_2 - L_2) \begin{matrix} > 90^\circ \\ < 270^\circ \end{matrix}$.

δ positive and $< 180^\circ$.

$$tg\tau = \frac{D}{R_2 . \sin(\lambda_3 - \lambda_1)} \quad \tau \text{ positive and } < 180^\circ \quad \ldots \quad \ldots \quad (7)$$

$$T = \frac{D}{\sin \tau} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (8)$$

$$tg\Sigma = \pm \frac{C}{T . \sin(\tau + \psi)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (9)$$

$$\sigma = \Sigma - \delta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

NOTE.—σ always $< 90^\circ$, positive or negative. Take Σ positive or negative, so that σ has always its least value :

$$S = \frac{C . \sin\delta}{\sin\Sigma} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (11)$$

$$P = \frac{t_1}{t_2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (12)$$

$$d = \frac{P + 1}{A + PB} S \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

$$tgu = \frac{1}{d \,.\, \cos \,.\, \sigma} \quad u \text{ positive and } < 180^\circ \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

$$tg\omega = \frac{tg\sigma \,.\, \cos u}{\sqrt{2} \,.\, \sin (45^\circ - u)} \quad \text{comp. log } \sqrt{2} = 9{\cdot}84949 - 10 \quad (15)$$

NOTE.—ω is always $<90^\circ$, positive or negative.

$$\Omega = \frac{d \,.\, \sin \sigma}{\sin \omega} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

$$h = \Omega(\mathrm{A} + \mathrm{PB})\mathrm{R}_2{}^3 \sin^3 \delta \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

$$j_1 = \frac{k^2}{6} t_1 (t_1 + 2t_2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

$$j_2 = \frac{k^2}{6} t_2 (2t_1 + t_2) \quad \log \frac{k^2}{6} = 5{\cdot}69301 - 10 \quad \ldots \quad \ldots \quad (19)$$

$$\mathfrak{M} = \frac{\mathrm{A}j_1 + \mathrm{PB}j_2}{h} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

3. *Solution of* $\mathfrak{M} \,.\, \sin^4 z = \sin (z + \omega)$.

4. *Calculation of* r_1 *and* r_3.—

$$r_2 = \frac{\mathrm{R}_2 \sin \delta}{\sin z} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (21)$$

$$\mathfrak{r}_2 = \frac{\mathrm{R}_2 \,.\, \cos \beta_2 \,.\, \sin (\delta - z)}{\sin z} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

$$\mathrm{Y}_1 = \frac{j_1}{r_2{}^3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (23)$$

$$\mathrm{Y}_2 = \frac{j_2}{r_2{}^3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (24)$$

$$n_1 = \frac{t_2}{t_1 + t_2} (1 + \mathrm{Y}_1) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (25)$$

$$n_2 = \frac{t_1}{t_1 + t_2} (1 + \mathrm{Y}_2) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (26)$$

$$\mathfrak{r}_1 = \left\{ \begin{array}{l} \mathfrak{r}_2 \,.\, \sin (\lambda_2 - \lambda_3) + n_1 \mathrm{R}_1 \sin (\mathrm{L}_1 - \lambda_3) \\ + n_2 \mathrm{R}_3 \sin (\mathrm{L}_3 - \lambda_3) - \mathrm{R}_2 \sin (\mathrm{L}_2 - \lambda_3) \end{array} \right\} : n_1 \sin (\lambda_1 - \lambda_3) \quad (27)$$

$$\mathfrak{r}_3 = \frac{\mathfrak{r}_2 tg\beta_2 - n_1 \mathfrak{r}_1 tg\beta_1}{n_2 tg\beta_3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (28)$$

$$ctg\delta_1 = \frac{\mathfrak{r}_1}{\mathrm{R}_1 \sin (\lambda_1 - \mathrm{L}_1)} - \cot (\lambda_1 - \mathrm{L}_1) \quad \ldots \quad \ldots \quad \ldots \quad (29)$$

$$l_1 = \lambda_1 + \delta_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (30)$$

$$ctg\delta_3 = \frac{\mathfrak{r}_3}{R_3 \sin(\lambda_3 - L_3)} - \cot(\lambda_3 - L_3) \quad \ldots \quad \ldots \quad \ldots \quad (31)$$

$$l_3 = \lambda_3 + \delta_3 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (32)$$

$$tg\, b_1 = \frac{\mathfrak{r}_1\, tg\beta_1 \sin(l_1 - \lambda_1)}{R_1 \sin(\lambda_1 - L_1)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (33)$$

$$tg\, b_3 = \frac{\mathfrak{r}_3\, tg\beta_3 \sin(l_3 - \lambda_3)}{R_3 \sin(\lambda_3 - L_3)} \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (34)$$

$$r_1 = \frac{\mathfrak{r}_1\, tg\beta_1}{\sin b_1} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (35)$$

$$r_3 = \frac{\mathfrak{r}_3\, tg\beta_3}{\sin b_3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (36)$$

5. *Calculation of Inclination and Ascending Node.*—

$$tg\, D = \frac{tgb_1 \,.\, \sin(l_3 - l_1)}{tgb_3 - tgb_1 \,.\, \cos(l_3 - l_1)} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (37)$$

$$\Omega = l_1 - D \quad D < 180^\circ \text{ or } > 180^\circ \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

Note whether the motion is direct or retrograde.

$$tg\, i = \frac{tgb_1}{\sin D} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

For direct motion $i < 90^\circ$; for retrograde $i > 90^\circ$.

6. *Calculation of Eccentricity and Major Axis.*—

$$\sin(v_3 \sim v_1) = \frac{\sin(l_3 \sim l_1) \,.\, \cos b_1 \,.\, \cos b_3}{\cos i} \quad \ldots \quad \ldots \quad \ldots \quad (40)$$

$$\sqrt{p'} = \frac{r_1 r_3 \sin(v_3 \sim v_1)}{k(t_1 + t_2)} \quad \log k = \bar{2}\cdot 23558 \quad \ldots \quad \ldots \quad (41)$$

$$\sqrt{p} = \left[1 + \frac{\sin^2(v_3 \sim v_1) \,.\, \sqrt{r_1 r_3}}{6p'}\right] \sqrt{p'} \quad \ldots \quad \ldots \quad (42)$$

$$tg\, v_1 = \cot(v_3 - v_1) - \frac{r_1(p - r_3)}{r_3(p - r_1)\sin(v_3 - v_1)} \quad \ldots \quad \ldots \quad (43)$$

$$e^2 = \frac{(p - r_1)(p - r_3)}{r_1 r_3 \cos v_1 \,.\, \cos v_3} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (44)$$

$$a = \frac{p}{1 - e^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (45)$$

$$T^d = a^{\frac{3}{2}} \times 365\cdot 256^d \qquad \log 365\cdot 256 = 2\cdot 56260 \quad (46)$$

$$m'' = \frac{1296\,000''}{T^d} \qquad \log 1296\,000 = 6\cdot 11260 \quad (47)$$

7. *Calculation of the Longitude of Perihelium and of Mean Longitude.*—

$$tgu_1 = \frac{tg\mathrm{D}}{\cos i} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (48)$$

When D $<180°$ $u_1 < 180°$; when D $>180°$ $u_1 > 180°$.

$$\Pi = u_1 + \Omega - v_1 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (49)$$

$$tg\,\frac{w}{2} = tg\,\frac{v_1}{2}\sqrt{\frac{1-e}{1+e}} \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (50)$$

$$\mathrm{M} = w - e\,.\sin w \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (51)$$

M and w to be taken in parts of the radius.

$$\tau^d = \mathrm{E}_1 - \mathrm{E}_0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (52)$$

$$\mathrm{L}'' = \tau^d m'' + \mathrm{M} \times 206\ 265''$$

$$\log 206\ 265 = 5{\cdot}31442 \quad \ldots \quad \ldots \quad (53)$$

$$\mathrm{L}_0 = \mathrm{L} + \Pi \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (54)$$

Calculation of Juno's Orbit.

1. *The Data.*—

$$t_1 = 11^{\mathrm{d}}{\cdot}96324 \qquad t_2 = 9^{\mathrm{d}}{\cdot}97119$$

$$\log \mathrm{R}_1 = 9{\cdot}99968 \qquad \log \mathrm{R}_2 = 9{\cdot}99810 \qquad \log \mathrm{R}_3 = 9{\cdot}99697$$

$$\mathrm{L}_1 = 192°\ 28'\ 28'' \qquad \mathrm{L}_2 = 204°\ 19'\ 49'' \qquad \mathrm{L}_3 = 214°\ 16'\ 10''$$

$$\lambda_1 = 354°\ 44'\ 32'' \qquad \lambda_2 = 352°\ 34'\ 22'' \qquad \lambda_3 = 351°\ 34'\ 30''$$

$$\beta_1 = -4°\ 59'\ 31'' \qquad \beta_2 = -6°\ 21'\ 55'' \qquad \beta_3 = -7°\ 17'\ 51''$$

2. *Preparatory Calculations.*—

$$\log \mathrm{P} = 0{\cdot}07910 \qquad t_1 + t_2 = 21^{\mathrm{d}}{\cdot}93443$$

$$\log t_1 = 1{\cdot}07785 \qquad \log t_2 = 0{\cdot}99875 \qquad \log (t_1 + t_2) = 1{\cdot}34113$$

$$\log (t_1 + 2t_2) = 1{\cdot}50387 \qquad \log (2t_1 + t_2) = 1{\cdot}53017$$

$$\lambda_1 - \mathrm{L}_1 = 162°\ 16'\ 4'' \qquad \lambda_1 - \mathrm{L}_2 = 150°\ 24'\ 43''$$
$$\lambda_1 - \mathrm{L}_3 = 140°\ 28'\ 22''$$

$$\lambda_3 - \mathrm{L}_1 = 159°\ 6'\ 2'' \qquad \lambda_3 - \mathrm{L}_2 = 147°\ 14'\ 41''$$
$$\lambda_3 - \mathrm{L}_3 = 137°\ 18'\ 20''$$

$$\lambda_3 - \lambda_1 = -3°\ 10'\ 2'' \qquad \lambda_3 - \lambda_2 = -\ 59'\ 52''$$
$$\lambda_2 - \lambda_1 = -\ 2°\ 10'\ 10'' \qquad \lambda_2 - \mathrm{L}_2 = 148°\ 14'\ 33''$$

$$\mathrm{L}_3 - \mathrm{L}_2 = 9°\ 56'\ 21'' \qquad \mathrm{L}_3 - \mathrm{L}_1 = 21°\ 47'\ 42''$$
$$\mathrm{L}_2 - \mathrm{L}_1 = 11°\ 51'\ 21''$$

3. *Preliminary Calculations for* A, B, C, *and* D.—

log $tg\beta_3$	9·10741 (n)	log $tg\beta_3$	9·10741 (n)	log $tg\beta_3$	9·10741 (n)
log sin $(\lambda_1 - L_1)$	9·48368	log sin $(\lambda_1 - L_3)$	9·80376	log sin $(\lambda_1 - L_2)$	9·69351
log (s_1)	8·59109 (n)	log (s_3)	8·91117 (n)	log (s_5)	8·80092 (n)
$(s_1) =$	− 0·039003	$(s_3) =$	−0·81502	$(s_5) =$	−0·063230
log $tg\beta_1$	8·94124 (n)	log $tg\beta_1$	8·94124 (n)	log $tg\beta_1$	8·94124 (n)
log sin $(\lambda_3 - L_1)$	9·55234	log sin $(\lambda_3 - L_3)$	9·83129	log sin $(\lambda_3 - L_2)$	9·73324
log (s_2)	8·49358 (n)	log (s_4)	8·77253 (n)	log (s_6)	8·67448 (n)
$(s_2) =$	−0·031159	$(s_4) =$	−0·059229	$(s_6) =$	−0·047259
$(s_1) - (s_2) =$	−0·007844	$(s_3) - (s_4) =$	−0·022273	$(s_5) - (s_6) =$	−0·015971
log $tg\beta_1$	8·94124 (n)	log $tg\beta_3$	9·10741 (n)	$(s_7) =$	0·073457
log cos $(\lambda_3 - L_2)$	9·92479 (n)	log cos $(\lambda_1 - L_2)$	9·93932 (n)	$(s_8) =$	0·111360
log (s_7)	8·86603	log (s_8)	9·04673	$(s_4) - (s_8) =$	−0·037903

4. *Calculation of* A, B, C, *and* D. (1) *to* (4).—

log R_1	9·99968	log R_3	9·99697	log R_2	9·99810	log R_2	9·99810
log $[(s_1)-(s_2)]$	7·89454 (n)	log $[(s_3)-(s_4)]$	8·34778 (n)	log $[(s_5)-(s_6)]$	8·20333 (n)	log $[(s_7)-(s_8)]$	8·57867 (n)
log A	7·89422 (n)	log B	8·34475 (n)	log C	8·20143 (n)	log D	8·57677 (n)
		log P	0·07910	A =	−0·0078382		
		log PB	8·42385 (n)	PB =	−0·0265370		
		log (A+PB) =	8·53624 (n)	A+PB =	−0·0343752		

5. *Calculations preceding the Equation of Gauss.* (5) *to* (20).—

log $tg\beta_2$	9·04748 (n)	log $tg(\lambda_2-L_2)$	9·79169 (n)	og D	8·57677 (n)
log sin (λ_2-L_2)	9·72125	log cos ψ	9·99046 (n)	c. log R_2	0·00190
log $tg\psi$	9·32623 (n)	log $tg\delta$	9·80123	c. log sin $(\lambda_3-\lambda_1)$	1·25766 (n)
$\psi=168°\ 2'$		$\delta=32°\ 19'\ 24''$	$\tau=34°\ 27'\ 2''$	log $tg\tau$	9·83633
log D	8·57677 (n)	log C	8·20143 (n)	$\Sigma=$	$31°\ 56'\ 10''$
log sin τ	9·75259	c. log T	1·17582 (n)	$\delta=$	$32°\ 19'\ 24''$
log T	8·82418 (n)	c. log sin $(\tau+\psi)$	0·41746 (n)	$\sigma=-$	$23'\ 14''$
$\tau+\psi=202°\ 29'\ 2''$		log $(tg\Sigma)$	9·79471 (n)	P+1 =	2·1998
log C	8·20143 (n)	log S	8·20611 (n)	log d	0·01225
log sin δ	9·72811	log (P+1)	0·34238	log cos σ	9·99999
c. log sin Σ	0·27657	c. log (A+PB)	1·46376 (n)	log cot u	0·01224
log S	8·20611 (n)	log d	0·01225	$u=$	$44°\ 11'\ 34''$
log $tg\sigma$	7·82984 (n)	log d	0·01225	$45°-u=$	$48'\ 26''$
log cos u	9·85552	log sin σ	7·82983 (n)	log Ω	8·46858
c. log $\sqrt{2}$	9·84949	c. log sin ω	0·62650 (n)	log (A+PB)	8·53624 (n)
c. log sin $(45°-u)$	1·85114	log Ω	8·46858	log R_2^3	9·99430
log $tg\omega$	9·38599 (n)	$\omega=-13°\ 40'\ 11''$		log $\sin^3\delta$	9·18433
				log h	6·18345 (n)

$\log \frac{k_2}{6}$	$5{\cdot}69301-10$	$\log \frac{k^2}{6}$	$5{\cdot}69301-10$
$\log t_1$	$1{\cdot}07785$	$\log t_2$	$0{\cdot}99875$
$\log (t_1+2t_2)$	$1{\cdot}50387$	$\log (2t_1+t_2)$	$1{\cdot}53017$
$\log j_1$	$8{\cdot}27473-10$	$\log j_2$	$8{\cdot}22193-10$

$\log A$	$7{\cdot}89422\ (n)$	$\log PB$	$8{\cdot}42385\ (n)$	(s_9)	$=-0{\cdot}00014755$
$\log j_1$	$8{\cdot}27473$	$\log j_2$	$8{\cdot}22193$	(s_{10})	$=-0{\cdot}00044236$
$\log (s_9)$	$6{\cdot}16895\ (n)$	$\log (s_{10})$	$6{\cdot}64578\ (n)$	$(s_9)+(s_{10})$	$=-0{\cdot}00058991$
		$\log [(s_9)+(s_{10})]$	$6{\cdot}77079\ (n)$		
		$\log h$	$6{\cdot}18345\ (n)$		
		$\log \mathfrak{M}$	$0{\cdot}58734$		

6. *Solution of the Equation of Gauss.*—

$\log \sin \omega$	$9{\cdot}374$	$\lambda = 0{\cdot}5$	$\log \sin z_2$	$9{\cdot}394$
	4			4
$\log \sin^4 \omega$	$7{\cdot}496$	$\mu = 10{\cdot}5$	$\log \sin^4 z_2$	$7{\cdot}576$
$\log \mathfrak{M}$	$0{\cdot}587$	$\mu - 4\lambda = 8{\cdot}5$	$\frac{\Delta}{\mu-4\lambda} =$	$9'\ 24''$
$\log \sin (z_2+\omega)$	$8{\cdot}083$	$\Delta = 80$		
$z_2+\omega = 41'\ 42''$		$\omega = -13^\circ\ 40'\ 11''$	$z_2 = 14^\circ\ 21'\ 53''$	
			$z_2 = 14^\circ\ 31'\ 17''$	

log sin z_3	9·39923	$\lambda = 0·8$	log sin z_4	9·39993
	4			4
log $\sin^4 z_3$	7·59692	$\mu = 14$	log $\sin^4 z$	7·59972
log $\mathfrak{M}$	0·58734	$\mu - 4\lambda = 10·8$	$\frac{\Delta}{\mu - 4\lambda} = 26''$	
log sin $(z_4 + \omega)$	8·18426	$\Delta = 280$		
$z_4 + \omega = 52' \ 33''$		$\omega = -13° \ 40' \ 11''$	$z_4 = 14° \ 32' \ 44''$	
Test.			$z_5 = 14° \ 33' \ 10''$	
log sin z_5	9·40014	$\lambda = 0·8$	log sin z_6	9·40014
	4			4
log $\sin^4 z_5$	7·60056	$\mu = 13·7$	log $\sin^4 z_6$	7·60056
log $\mathfrak{M}$	0·58734	$\mu - 4\lambda = 10·5$	$\frac{\Delta}{\mu - 4\lambda} =$	$0''$
log sin $(z_6 + \omega)$	8·18790	$\Delta = 0$		
$z_6 + \omega = 52' \ 59''$		$\omega = -13° \ 40' \ 11'$	$z_6 = 14° \ 33' \ 10'$	

7. *Calculation of* r_1 *and* r_3. (21) *to* (28).—

log R_2	9·99810	$\delta = 32° \ 19' \ 24''$	log R_2	9·99810
log sin δ	9·72811	$z = 14 \ \ 33 \ \ 10$	log cos β_2	9·99731
c. log sin z	0·59986	$\delta - z = 17° \ 45' \ 14'$	c. log sin z	0·59986
log r_2	0·32607	$r_2 = 2·1187$	log sin $(\delta - z)$	9·48459
$1 + Y_1 = 1·001979$		$1 + Y_2 = 1·001753$	log r_2	0·07986

$\log j_1$	8·27473	$\log t_2$	0·99875	$\log j_2$	8·22193	$\log t_1$	1·07785
$\log r_2^3$	0·97821	$\text{c.} \log (t_1 + t_2)$	8·65887	$\log r^3_2$	0·97821	$\text{c.} \log (t_1 + t^2)$	8·65887
$\log Y_1$	7·29652	$\log (1 + Y_1)$	0·00086	$\log Y_2$	7·24372	$\log (1 + Y_2)$	0·00076
$Y_1 = 0·001979$		$\log n_1$	9·65848	$Y_2 = 0·001753$		$\log n_2$	9·73748

$\log r_2$	0·07986	$\log R_2$	9·99810
$\log \sin (\lambda_2 - \lambda_3)$	8·24090	$\log \sin (L_2 - \lambda_3)$	9·73324 (n)
$\log (s_{11})$	8·32076	$\log (s_{14})$	9·73134 (n)
$\log n_1$	9·65848	$\log (n_2)$	9·73748
$\log R_1$	9·99968	$\log R_3$	9·99697
$\log \sin (L_1 - \lambda_3)$	9·55234 (n)	$\log \sin (L_3 - \lambda_3)$	9·83129 (n)
$\log (s_{12})$	9·21050 (n)	$\log (s_{13})$	9·56574 (n)
		$\log (S)$	8·46750
		$\text{c.} \log n_1$	0·34152
		$\text{c.} \log \sin (\lambda_1 - \lambda_3)$	1·25766
		$\log r_1$	0·06668

$(s_{11}) = 0·020929$
$(s_{12}) = -0·162366$
$(s_{13}) = -0·367908$
$(s_{14}) = 0·538688$
$(S) = 0·029343$

$\log r_2$	0·07986	$\log n_1$	9·65848
$\log tg\beta_2$	9·04748 (n)	$\log r_1$	0·06668
$\log (s_{15})$	9·12734 (n)	$\log tgB_1$	8·94124 (n)
$\log (S_1)$	8·94294 (n)	$\log (s_{16})$	8·66640 (n)

$(s_{15}) = -0·134074$
$-(s_{16}) = 0·046387$
$(S_1) = -0·087687$

M

c. log n_2	0·26252	l_1	$= 362° \; 55' \; 13''$
c. log $tg\beta_3$	0·89259 (n)	λ_1	$= 354° \; 44' \; 32''$
log r_3	0·09805	$l_1 - \lambda_1$	$= 8° \; 10' \; 41''$
		l_3	$= 370° \; 20' \; 12''$
		λ_3	$= 351° \; 34' \; 30''$
		$l_3 - \lambda_3$	$= 18° \; 45' \; 42''$

8. *Calculation of Heliocentric Coordinates.* (29) *to* (36).—

log r_1	0·06668	log r_3	0·09805
c. log R_1	0·00032	c. log R_3	0·00303
c. log sin $(\lambda_1 - L_1)$	0·51632	c. log sin $(\lambda_3 \quad L_3)$	0·16871
log (s_{17})	0·58332	log (s_{18})	0·26979

(s_{17})	3·83109	$\lambda_1 =$	$354° \; 44' \; 32''$
$-\cot(\lambda_1 - L_1)$	3·12734	$\delta_1 =$	$8° \; 10' \; 41''$
cot δ_1	6·95843	$l_1 =$	$2° \; 55' \; 13''$
(s_{18})	1·86117	$\lambda_3 =$	$351° \; 34' \; 30''$
$-\cot(\lambda_3 - L_3)$	1·08390	$\delta_3 =$	$18° \; 45' \; 42''$
cot δ_3	2·94507	$l_3 =$	$10° \; 20' \; 12''$

log r_1	0·06668	log r_1	0·06668
log $tg\beta_1$	8·94124 (n)	log $tg\beta_1$	8·94124 (n)
log sin $(l_1 - \lambda_1)$	9·15304	c. log sin b_1	1·32292 (n)
c. log R_1	0·00032	log r_1	0·33084
c. log sin $(\lambda_1 - L_1)$	0·51632		
	8·67760		
log tgb_1		(n) $b_1 =$	$-2° \; 43' \; 30''$

log r_3	0·09805	log r_3	0·09805
log $tg\beta_3$	9·10741 (n)	log $tg\beta_3$	9·10741
log sin $(l_3 - \lambda_3)$	9·50736	c. log sin b_3	1·11665 (n)
c. log R_3	0·00303	log r_3	0·32211
c. log sin $(\lambda_3 - L_3)$	0·16871		
	8·88456		
log tgb_3		(n) $b_3 =$	$-4° \; 23' \; 3''$

Motion direct. $i < 90°$.

9. *Calculation of Inclination and Ascending Node.* (37) *to* (39).—

$ltgb_1$	8·67760(n)	$tgb_3 =$	$-0·076665$	$l_3 =$	10° 20′ 12″
$l\cos(l_3-l_1)$	9·99635	$-(s_{18}) =$	0·047201	$l_1 =$	2 55 13
$l(s_{18})$	8·67395(n)	$(s_{19}) =$	$-0·029464$	$l_3-l_1 =$	7 24 59

$ltgb_1$	8·67760(n)	$ltgb_1$	8·67760(n)	$l_1 =$	2° 55′ 13″ + 360°
$l\sin(l_3-l_1)$	9·11085	$l\sin D$	9·30993(n)	D =	11 46 44 + 180
c. $l(s_{19})$	1·53071(n)	$ltgi$	9·36767	☊ =	171 8 29
ltgD	9·31916		$i = 13° 7' 30''$		

10. *Calculation of Eccentricity and Major Axis.* (40) *to* (47).—

log sin $(l_3 \sim l_1)$	9·11085	log sin $(v_3 \sim v_1)$	9·12058
log cos b_1	9·99951	log r_1	0·33084
log cos b_3	9·99873	log r_3	0·32211
c. log cos i	0·01149	c. log k	1·76442
log sin $(v_3 \sim v_1)$	9·12058	c. log (t_1+t_2)	8·65887
log p'	0·39364	log $\sqrt{p'}$	0·19682
log 6	0·77815	$v_3 \sim v_1 = 7° 35' 7''$	
log $6p'$	1·17169		

log sin² $(v_3 \sim v_1)$	8·24116			log $(1+(s_{20}))$	0·00108
log $\sqrt{r_1}$	0·16542			log $\sqrt{p'}$	0·19682
log $\sqrt{r_3}$	0·16105			log $\sqrt{p}$	0·19790
c. log $6p'$	8·82831	$p =$	2·48768	log p	0·39580
log (s_{20})	7·39594	$(s_{20}) =$	0·0024885	$1+(s_{20}) =$	1·00249

$p = 2·48768$	$p = 2·48768$
$r_1 = 2·14210$	$r_3 = 2·09948$
$p-r_1 = 0·34558$	$p-r_3 = 0·38820$

log $(p-r_3)$	9·58906		
log r_1	0·33084	$-(s_{21}) =$	$-8·68280$
c. log r_3	9·67789	cot $(v_2-v_1) =$	7·52104
c. log $(p-r_1)$	0·46145	$tg\ v_1 =$	$-1·16176$

c. log sin (u_3-u_1)	0·87942	$v_1 = -49°\ 16'\ 46''$	
log (s_{21})	0·93866	$\frac{v_1}{2} = -24\ 38\ 23$	

log $(p-r_1)$	9·53855	$v_1 = -49°\ 16'\ 46''$	
log $(p-r_3)$	9·58906	$v_3-v_1 = 7\ 35\ 7$	
c. log r_1	9·66916	$u_3 = -41\ 41\ 39$	
c. log r_3	9·67789	log p	0·39580
c. log cos v_1	0·18542	l. $\cos^2\phi$	9·97256
c. log cos v_3	0·12686	log a	0·42324
log e^2	8·78694	$e = 0·24744$	
log $e = l \sin\phi = 9·39347$		$\phi = 14°\ 19'\ 33''$	

log $a^{\frac{3}{2}}$	0·63486	log 1·296000	6·11260
log $365^{d}·256$	2·56260	log T^d	3·19746
log T^d	3·19746	log m'' 2·91514	
$T^d = 1575^{d}·65$		$m'' = 822''·5$	

11. *Calculation of Longitude of Perihelium and Mean Longitude.* (48) *to* (54).—

log tgD	9·31916	$u_1 =$	$192°\ 5'\ 8''$
log cos i	9·98851	☊ $=$	$171\ 8\ 29$
log tgu_1	9·33065		$363\ 13\ 37$
$1+e = 1·24744$		$-v_1 =$	$49\ 16\ 46$
$1-e = 0·75256$		$\Pi =$	$52\ 30\ 23$

log $tg\ \frac{v}{2}$	9·66151(n)	log e	9·39347	$w =$	$-0·684489$
log $\sqrt{1-e}$	9·93827	log sin w	9·80091(n)	$-(s_{22}) =$	0·156452
c. log $\sqrt{1+e}$	9·95199	log (s_{22})	9·19438(n)	M $=$	$-0·528037$
log $tg\ \frac{w}{2}$	9·55177(n)	$\frac{w}{2} = -19°\ 36'\ 33''$		$w = -39°\ 13'\ 6''$	

$E_1 =$	1804, Oct. 5·458644 log M.	9·72263(*n*)
$E_0 =$	1805, Jan. 0·000000 log 206265″	5·31442
$\tau^d =$	86^d·541356 log(s_{23}	5·03705(*n*)

log τ^d	1·93722	$(s_{23}) =$	$-108905''$	L =	$-10° 28' 45''$
log m''	2·91514	$(s_{24}) =$	71180	Π =	52 30 23
log (s_{24})	4·85236	L″ =	− 37725	$L_0 =$	41 58 22

Syra, Greece: 1903 *August* 20.

Ephemeris for Physical Observations of Saturn, 1903–4.
By A. C. D. Crommelin.

The discovery by Professor Barnard of a conspicuous white spot on *Saturn* last July has reawakened interest in the question of the planet's rotation, and has illustrated the remarkable variation of rate that prevails in different latitudes. At Mr. Denning's suggestion I have computed an ephemeris giving the longitudes of central meridian and the times of transit of the zero meridian on two assumptions as to the rotation period, viz.:

System I. $10^h\ 14^m{\cdot}4$ for equatorial spots (Hall).

System II. $10^h\ 37^m{\cdot}92$ for the spot discovered by Barnard last July (period provisionally deduced by Denning).

The quantities corresponding to P, B, *B* in the *Jupiter* ephemeris may be taken from the *Nautical Almanac* ephemeris of the ring, or more conveniently from the *Connaissance des Temps*, as this is fuller and contains the longitude of the Earth in the plane of the ring. In order to utilise the data of the *Connaissance* without interpolation, I give the longitude of the central meridian for Paris midnight, corresponding to $11^h\ 50^m{\cdot}65$ G.M.T.; but the transits of zero meridian are given for G.M.T. The quantities are to be interpolated for the time for which they are required, the equation of light having been already applied. The approximate increase of the longitude of central meridian in five days is 4219° in System I., 4063° in System II. I have commenced a new system of zero meridians, as Mr. Marth used a different period of rotation ($10^h\ 13^m{\cdot}6$) in his ephemerides, of which the last appeared in vol. lv. p. 164. The small figures between the transits of zero meridians denote the number of intervening rotations, and the table at the end facilitates the determination of the times of intervening transits.